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THE FUNDAMENTAL THEOREM OF CELESTIAL MECHANICS.

BY J. L. COOLIDGE.

We mean by this sonorous title the classical theorem which tells us that the center of gravity of a planet traces with regard to the sun, a conic having one focus at the sun's center. The following proof is offered in the hope that it may be found simpler than some of those in current use.

We start with the Newtonian law of universal gravitation and give the usual proof that since we have a central force the point must trace a plane curve. The plane of this curve we take as the z plane, place the origin at the center of the sun, and write the familiar equations of motion

$$x'' = \frac{kx}{r^3}, \qquad y'' = \frac{ky}{r^3}, \tag{1}$$

$$xy'' - yx'' = 0,$$

$$xy' - yx' = C.$$

At this point, instead of seeking immediately for the curve we want, let us rather look for its polar reciprocal in a unit circle with center at the origin. The equation of a tangent to our original curve is

$$Xy' - Yx' = C.$$

Its pole in the unit circle is

$$X = \frac{y'}{C} \qquad Y = -\frac{x'}{C}.$$

We seek the curvature of this polar curve

$$X' = \frac{y''}{C} = \frac{ky}{Cr^3}, \qquad Y' = -\frac{x''}{C} = \frac{-kx}{Cr^3},$$

$$X'' = \frac{k}{C}\,\frac{(ry' - 3yr')}{r^4}, \qquad Y'' = \frac{-k}{C}\,\frac{(rx' - 3xr')}{r^4},$$

$$\frac{X'Y'' - Y'X''}{(X'^2 + Y'^2)^{3/2}} = \frac{k^2}{C^2}\,\frac{(xy' - yx')}{r^6} \div \frac{k^3}{C^3}\left(\frac{x^2 + y^2}{r^6}\right)^{3/2} = \frac{C^2}{k}.$$

Since this polar curve has a constant curvature it is a circle. Hence the original curve was a conic with a focus at the origin.

AMERICAN EXPEDITIONARY FORCES,
October, 1918.